TD INF567

WiFi Performance Analysis

Version: 20 Jan. 2021

1 System Model

We consider a set of $n > 1$ stations communicating with IEEE 802.11. We assume that all nodes are in (radio) visibility of each other (i.e., there are no hidden or exposed nodes, they can all hear each other) and that transmission queues are saturated (i.e., stations have always a packet to transmit).

Recall that in the IEEE 802.11 standard, stations draw a random back-off timer before transmitting. Back-off durations are computed as a multiple of a standardized time interval called a *slot*. The back-off timer is decremented whenever a slot is sensed idle by the station, i.e., when there is no transmission. When a station completes its back-off, it attempts to reserve the channel by sending a RTS packet. If no other station has completed its back-off on the same slot, the handshake RTS-CTS-Data-ACK completes successfully. If two RTS collide, the collision ends after the transmission of the RTSs. At the $(K+1)$th attempt, either a packet is successfully transmitted or discarded. During an activity period (either a successful transmission or a RTS collision), all stations freeze their back-off timers. At the end of an activity period, every station waits for a short period DIFS before decrementing its back-off timer again. We assume that all stations have the same back-off parameters.

Question 1 *Show that the time spent in back-off up to a time t is the same for all stations.*

This observation allows us to "forget" activity periods and to study back-off processes conditionally to the fact that stations are in back-off. We call *back-off time* this conditional time.

2 Back-off Analysis

Let $\beta \in [0; 1]$ be the station attempt rate per slot (or equivalently the probability of attempting transmission on a given slot). Let $\gamma \in [0; 1]$ be the collision probability, i.e., the probability that a collision occurs in the network. By symmetry, all stations have the same attempt rate and see the same collision probability.

We assume that the number of attempts on a slot is a binomial random variable with parameters n and β: $\mathbb{P}[k \text{ attempts}] = \binom{n}{k}\beta^k(1-\beta)^{n-k}$. This assumption is called the *decoupling approximation* because it states that the attempt probability is independent on the back-off processes.

Let b_k be the average back-off duration at the kth attempt for a packet ($k = 0, ..., K$). For a packet, let R be the number of attempts until success or discard, and X the time needed to successfully transmit it.

Question 2 *Show that* $\mathbb{E}[R] = \sum_{k=0}^{K} \gamma^k$ *and* $\mathbb{E}[X] = \sum_{k=0}^{K} b_k \gamma^k$.

Definition 1 *A random point process on the positive real line* $\psi = \{t_n, n \geq 1\}$ *such that* $0 < t_1 < t_2 < ...$ *and* $t_n \to +\infty$ *when* $n \to +\infty$ *and such that the inter-arrival times* $X_n = t_n - t_{n-1}, n \geq 1$ *are i.i.d random variables is called a renewal process.*

Theorem 1 (Renewal reward) *Assume a renewal process* $\psi = \{t_n, n \geq 1\}$ *in which a reward* R_j *is earned during cycle length* X_j *and such that* (R_j, X_j), $j \geq 1$ *are i.i.d. with* $\mathbb{E}|R_j| < \infty$. *Denote* $N(t)$ *its counting process, i.e.,* $N(t) = \sup\{n, t_n \leq t\}$ *and* $R(t) = \sum_{j=1}^{N(t)} R_j$ *the cumulative reward up to* t. *Then, the long run rate at which reward is earned is given by:*

$$\lim_{t \to \infty} \frac{R(t)}{t} = \frac{\mathbb{E}[R]}{\mathbb{E}[X]}.$$

Question 3 *Express* β *as a function* $G(\gamma)$ *of* γ.

In the IEEE 802.11 standard, there is a contention window $CW_{\min}$, and the contention window CW is doubled at every collision. After m collisions, the contention window remains constant. For a contention window CW, the back-off timer is uniformly drawn in $[0, 1, ..., CW - 1]$.

Question 4 *Compute the* b_k, $k = 0, ..., K$ *in this case.*

3 Fixed Point

We now intend to express the collision probability as a function $\Gamma(\beta)$ of the attempt rate.

Question 5 *Assume that a station attempts an access. Using the decoupling assumption, give the probability of collision for this station as a function of* n *and* β.

We can now characterize the system at equilibrium by a fixed point equation:

$$\gamma = \Gamma \circ G(\gamma).$$

Theorem 2 (Brouwer) *Every continuous function from a convex compact subset of an Euclidean space to itself has a fixed point.*

Question 6 *Show the existence of a fixed point.*

By taking the derivative of G with respect to γ, we can show that G is a decreasing function of γ if $\{b_k\}_{0\leq k\leq K}$ is an increasing sequence (the calculation is a bit tedious and thus omitted).

Question 7 *Show that $\Gamma(G(\gamma))$ has a unique fixed point if $\{b_k\}_{0\leq k\leq K}$ is an increasing sequence.*

4 Throughput Calculation

We now want to deduce from the previous analysis, the saturated throughput of the network. At every slot, the network may be in either of the three possible states: 1) the slot is idle because none of the stations is transmitting or attempting an access; 2) one station is currently transmitting successfully; 3) a collision is on going because at least two stations have attempted an access on the same slot. Let σ be the slot duration. Assume that all stations transmit packets of length L at physical data rate C. We denote T_c the transmission time of a RTS plus DIFS, i.e., the time spent in a collision. We denote T_s the time due to the RTS-CTS-Data-ACK handshake plus DIFS. Note that the ends of successful transmissions or collisions are renewal instants. The reward is the number of bits successfully transmitted. The cycle length can be decomposed as follows: the average duration until an attempt plus the average time of activity (i.e. transmission or collision).

Question 8 *Show that the average reward between two renewal instants is*

$$\frac{n\beta(1-\beta)^{n-1}L}{1-(1-\beta)^n}.$$

Question 9 *What is the average duration until an attempt, as a function of σ, β and n?*

Question 10 *What is the average duration of an activity period (successful transmission or collision) between two renewal instants?*

Question 11 *Deduce from the previous questions and the renewal reward theorem the saturated throughput S for the network.*

Question 12 *Numerical work: Draw the normalized saturated throughput S/C as a function of n with $K = 10$, $m = 6$, $\sigma = 50$ μs, $L = 8584$ bits, $ACK = 240$ bits, $SIFS = 28$ μs, $DIFS = 128$ μs, $CWmin = 32$, $C = 1$ Mbps. How do you explain the small increase of the curve for very low n?*